ECON 3510 - Intermediate Macroeconomic Theory

Fall 2015

Mankiw, *Macroeconomics, 8th ed.*, Chapter 3

Chapter 3: A Theory of National Income

Key points:

- Understand the aggregate production function
- Understand "crowding out" of investment
- Real vs. nominal interest rates, the Fisher Equation
- Know how to do comparative statics

The Firm:

- Firms make stuff from inputs
- Inputs:
  - capital, K
  - labor, L
- Firm's production technology given by a production function
  - A function of K and L
  - Production function: $Y = F(K, L)$
  - Properties:
    1. Positive Marginal Product: more capital $\Rightarrow$ more output $\rightarrow \frac{\partial F(K,L)}{\partial K} > 0$, $\frac{\partial F(K,L)}{\partial L} > 0$
    2. Diminishing Marginal Product: the additional amount of output you get for an increase in inputs decreases as use more of input, $\frac{\partial^2 F(K,L)}{\partial^2 K} < 0$, $\frac{\partial^2 F(K,L)}{\partial^2 L} < 0$
    - ∗ DRAW a concave production function. Note positive slope and decline in slope as K or L increases
    - ∗ Many we look at here will also have constant returns to scale (CRS): $\rightarrow zY = F(zK, zL)$ (this means if you increase inputs by a certain factor, output increases by the same factor)
- The objective of firms is to maximize profits
  - $\underbrace{\pi(K,L)}_{\text{profit}} = \underbrace{P}_{\text{price of output}} \times Y - (\underbrace{W}_{\text{wage}} \times L) - (\underbrace{R}_{\text{rental price for capital}} \times K) = P \times F(K,L) - (W \times L) - (R \times K)$
- Firms are competitive, meaning takes prices (P, W, R) as given
  - So the problem of the firm is to choose capital and labor to maximize profits
- Solve using calculus:
  1. $\frac{\partial \pi(K,L)}{\partial K} = 0$
  2. $\frac{\partial \pi(K,L)}{\partial L} = 0$

- Both these conditions must hold (they are necessary conditions of optimality) and if there are diminishing marginal products, we know we've found a max at this point (b/c the second derivative of the production function is negative - see calc book)
- Using our profit function, this means:
  1. $\frac{\partial \pi(K,L)}{\partial K} = P\frac{\partial F(K,L)}{\partial K} - R = 0$
  2. $\frac{\partial \pi(K,L)}{\partial L} = P\frac{\partial F(K,L)}{\partial L} - W = 0$
  * Equations 1 and 2 give demand for capital and labor:
  * $\rightarrow K^*$ solves $P \times MPK(K,L) = R$ or $MPK(K,L) = \frac{R}{P}$
  * $\rightarrow L^*$ solves $P \times MPL(K,L) = W$ or $MPL(K,L) = \frac{W}{P}$
  * What this means: hire capital until the marginal product of capital equals the marginal cost (the rental rate). That is, until the number of goods produced from another unit of capital equals the amount of goods that must be paid to use that capital. (Same for labor).
- What are the real economic profits?
  * $\pi = P \times Y - WL - RK$
  * plug-in solution for eq'm wage and rental rate: $\pi = P\times Y - (P \times MPL \times L) - (P \times MPK \times K)$
  * $\pi = P[Y - (MPL \times L) - (MPK \times K)]$
  * $\pi = P\underbrace{[F(K,L) - (MPL \times L) - (MPK \times K)]}_{=0,\text{ if } F(K,L) \text{ exhibits constant returns to scale}}$
  * The proof uses Euler's Theorem, see Mankiw, page 57, gist of it is that with CRS, $F(K,L) = (MPL \times L) + (MPK \times K)$
  * Proof: with CRS, $zY = F(zK, zL)$, thus, differentiating both sides by z implies: $\frac{\partial zY}{z} = Y = \frac{\partial F(zK,zL)}{\partial K}\frac{\partial zK}{\partial z} + \frac{\partial F(zK,zL)}{\partial L}\frac{\partial zL}{\partial z} = \frac{\partial F(zK,zL)}{\partial K}K + \frac{\partial F(zK,zL)}{\partial L}L$, now evaluate at $z = 1$ and you have $Y = K \times MPK + L \times MPL$
  * Thus if CRS, then economic profits are zero
  * Accounting profits = Econ profits + $(MPK \times K)$ - b/c don't take into account the opportunity cost of capital

The Aggregate Production Function

- Pretend the economy acts like one big firm (or, equivalently, is made up of identical competitive firms)
- $GDP = Y = F(K,L)$
- As with each firm, all output divided between capital and labor (i.e., economic profits are zero)
  - Capital gets $r \times K = MPK \times K$
  - Labor gets $w \times L = MPL \times L$
  - $Y = (MPK \times K) + (MPL \times L)$ b/c of CRS
- SHOW graph of labor and capital shares in U.S. over time...

Cobb-Douglas Production Function

- Common form for production function in economic models
- We use it for a couple main reasons:
  1. It's easy to work with
  2. Has been shown to do a pretty good job capturing the data

  - Constant labor and capital shares
  - Elasticity of substitution between capital and labor of about 1.

- Also, it can have constant returns to scale
- Has constant "factor shares"
  - shares of output paid to factors of production are the same regardless of the total amount of production
- Of the form: $Y = AK^{\alpha}L^{1-\alpha}$
  - A = technology (or TFP) - makes K and L more productive
  - α = capital's share of output
  - $1-\alpha$ = labor's share of output (note how we wrote this means it's CRS ($\alpha + (1-\alpha) = 1$))
  - See this:
    * $MPK = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^{1-\alpha} = \frac{\alpha Y}{K}$
    * $\Rightarrow MPK \times K = \alpha AK^{\alpha-1}L^{1-\alpha}K = \alpha AK^{\alpha}L^{1-\alpha} = \alpha Y$
    * Similarly, $MPL = (1-\alpha)\frac{Y}{L}$, thus labor's share, $MPL \times L = (1-\alpha)Y$
    * Thus: $MPK \times K + MPL \times L = \alpha Y + (1-\alpha)Y = Y$, which means function is CRS
- Factor prices:
  - Real wage = $\frac{W}{P} = MPL = (1-\alpha)\frac{Y}{L}$
  - Real rental rate = $\frac{R}{P} = MPK = \alpha\frac{Y}{K}$

Accounting ID as supply and demand:

- Now recall our National Accounts Identity: $Y = C + I + G + NX$
- LHS is supply of goods/services, RHS is demand

Aggregate Supply:

- $Y = F(K, L)$
- Assume (for now) that factor supply is fixed:
  - $K = \bar{K}$
  - $L = \bar{L}$
  - $\Rightarrow$ that MPL and MPK are fixed - as are factor prices
  - $\Rightarrow$ supply is fixed: $\bar{Y} = F(\bar{K}, \bar{L})$

Demand:

- Consumption
  - Consumption function: $C = C(Y - T)$
    * T= net taxes = taxes - transfers
    * $Y - T$ = disposable (after-tax) income

  - Marginal propensity to consume
    * MPC = how much of the next dollar do you spend?
    * MPC = $\frac{\partial C(Y-T)}{\partial (Y-T)}$

- <u>Investment</u>
  - $I = I(r)$, where r is the real interest rate
  - Fisher equation: real interest rate = nominal interest rate - inflation; $r = i - \pi$
  - SHOW graph of real and nominal rates together
  - $r \neq \frac{R}{P}$, r includes housing and inventories
  - I is plant and equipment, housing, inventories
  - as $r \uparrow$, $I \downarrow$; $\frac{\partial I(r)}{\partial r} < 0$

- <u>Government Spending</u>
  - assume $\bar{G}$, $\bar{T}$ fixed
  - Budget deficit if $\bar{G} > \bar{T}$
  - Budget surplus if $\bar{G} < \bar{T}$
  - Balanced budget if $\bar{G} = \bar{T}$

- <u>Net Exports</u>
  - Assume (for now) that these = 0; i.e. a "closed economy" model
  - Chap 6 will deal with an open economy

<u>Equilibrium in market for goods and services</u>:

- Supply = Demand
- $\underbrace{Y}_{\text{Supply}} = \underbrace{C + I + G}_{\text{Demand}}$
- $\rightarrow \bar{Y} = F(\bar{K}, \bar{L})$
- $\rightarrow \bar{C} = C(\bar{Y} - \bar{T})$
- $\rightarrow I = I(r)$
- $\bar{G}, \bar{T}$
- Note: Supply fixed, all aspect of demand, except investment are fixed
  - This is all by assumption to make solving the model easier
- Eq'm $\Rightarrow$
  - Eq'm implies that supply = demand
  - $F(\bar{K}, \bar{L}) = C[F(\bar{K}, \bar{L}) - \bar{T}] + I(r) + \bar{G}$ - one equation and one unknown - need to solve for r
  - Thus, if supply > demand $\Rightarrow r \downarrow \Rightarrow I \uparrow$
  - If supply < demand $\Rightarrow r \uparrow \Rightarrow I \downarrow$

<u>Another way to look at it: Eq'm in the loanable funds market</u>:

- Supply = demand
- Rearrange National Accounts Identify and get:
  - $I = Y - C - G = S$, where S is national savings
  - $I = \underbrace{(Y - T - C)}_{\text{private savings}} + \underbrace{(T - G)}_{\text{public savings}} = S$
  - $I(r) = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} = \bar{S}$
  - $I(r) = \bar{S}$, $\rightarrow r$ adjusts investment demand to equal supply (savings)
- DRAW the loanable funds market: I,S on horiz axis, r on vertical, $\bar{S}$ fixed, I(r) downward sloping...
- Dfn: exogenous variables: those determined outside the model
- Dfn: endogenous variables: those determined within the model
- Here:
  - exogenous variables: $\bar{Y}$, $\bar{K}$, $\bar{L}$, $\bar{C}$, $\bar{G}$, $\bar{T}$
  - endogenous variables: r, I

Comparative statics:

- Comparative statics are a tool to see how endogenous variables change when exogenous variables change
- $\frac{\partial \text{Endogenous Var.}}{\partial \text{Exogenous Var.}}$
- e.g., $\bar{G} \uparrow \Rightarrow \bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} \downarrow$
  - national savings falls when gov't spending increases
  - DRAW loanable funds market where national savings shifts to the left... highlight rise in interest rates
  - $\frac{\partial r^*}{\partial G} > 0$
  - $\frac{\partial I(r^*)}{\partial G} < 0 \rightarrow$ "crowding out"
    - Gov't spending crowds out private investment
  - SHOW UK interest rates with military spending graphs
  - SHOW US interest rates with recent gov't spending - no relation. Why? Fed? US has reserve currency for rest of world? Not sure.

A more complicated/realistic savings function:

- Note that savings may depend on the interest rate - you want to save more if you get a higher return
- Draw upward sloping savings $S(r)$ function
- Show shift in savings - still get crowing out, but not as severe since interest rate rising moves you along the savings supply curve
- May also want to note shifts in investment curve - e.g. from lower taxes on investment (determined outside the model (i.e., exogenous), so shift, not move along curve). Invest now higher at each interest rate. Increase in invest pushes interest rate up to restore eq'm.